The square root of 2 ain't rational

A Casual Talk By

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Some centuries B.C.

A simple assumption

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Its consequences

- So what?
- $a^2 = 2b^2$
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The problem

- And but so we said a and b have no common factor.
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All fractions are reducible

- Suppose $\frac{c}{d}$ is a rational number. If c and d have no common factor, then $a = b$ and $b = d$. If they have a common factor, divide both by their greatest common divisor. The result is $\frac{a}{b}$, with no common factor. ◀ Back

An even square has an even root

An even number, by definition, is expressible in the form $2k$, where k is any integer. On the other hand, an odd number is expressible by

$$2k + 1$$

Thus the square of an odd number is

$$(2k + 1)^2$$

i.e.

$$4k^2 + 4k + 1$$

i.e.

$$2 \times 2(k^2 + k) + 1$$

which is of the form $2k + 1$ with $2(k^2 + k)$ as k. Hence, an odd number produces an odd square, and thus if a square is even its root is even too. ◀ Back